ANALİZ II.2 QUİZ 2 ÇÖZÜMLERİ

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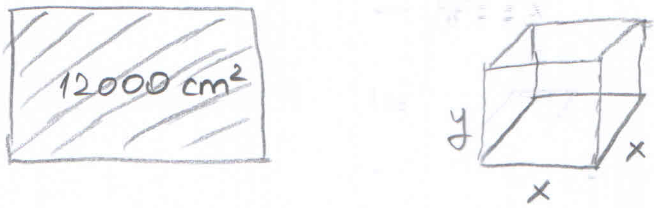
1) 12000 cm^2 lik bir malzemeden tabanı kare üstü açık bir kutu yapılmak isteniyor. En büyük hacimli kutunun boyutları ne olur?

Çözüm:



Kutunun yüzey alanı : $4xy + x^2 = 12000$

$$y = \frac{12000 - x^2}{4x}$$

Kutunun hacmi: $V = x^2 \cdot y$

$$V(x) = x^2 \cdot \frac{12000 - x^2}{4x} = 3000x - \frac{x^3}{4}$$

$$V'(x) = 3000 - \frac{3x^2}{4}$$

$$V'(x) = 0 \iff \frac{3x^2}{4} = 3000 \iff x^2 = 4000$$

$$\boxed{\begin{array}{l} x = 20\sqrt{10} \\ y = 10\sqrt{10} \end{array}}$$

2) a) $\lim\limits_{x \to 0^+} (\sin x)^{\frac{3}{\ln x}} = ?$ b) $\lim\limits_{x \to 0} \left(\operatorname{cosec} x - \frac{1}{x}\right) = ?$

Çözüm: a) 0^0, $y = (\sin x)^{\frac{3}{\ln x}} \Rightarrow \ln y = \frac{3}{\ln x} \cdot \ln(\sin x)$

$$\lim_{x \to 0^+} \ln y = \lim_{x \to 0^+} \frac{3\ln(\sin x)}{\ln x} = 3\lim_{x \to 0^+} \frac{\frac{\cos x}{\sin x}}{\frac{1}{x}} = 3\lim_{x \to 0^+} \cos x \cdot \frac{x}{\sin x} = 3 \cdot 1 \cdot 1 = 3$$

$$\lim_{x \to 0^+} y = e^{\lim\limits_{x \to 0^+} \ln y} = e^3$$

b) $$\lim_{x \to 0} \left(\operatorname{cosec} x - \frac{1}{x}\right) \overset{(\infty - \infty)}{=} \lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x}\right) = \lim_{x \to 0} \frac{x - \sin x}{x \sin x}$$

$$= \lim_{x \to 0} \frac{1 - \cos x}{\sin x + x\cos x} = \lim_{x \to 0} \frac{\sin x}{\cos x + \cos x - x\sin x}$$

$$= \lim_{x \to 0} \frac{\sin x}{2\cos x - x\sin x} = 0$$

3-) $f(x)=\dfrac{e^x}{x}$ fonksiyonunun grafiğini çiziniz.

Çözüm: $D_f = \mathbb{R}-\{0\}$

$$\lim_{x\to 0^+}\frac{e^x}{x}=+\infty \quad , \quad \lim_{x\to 0^-}\frac{e^x}{x}=-\infty$$

$$\lim_{x\to \infty}\frac{e^x}{x}=\lim_{x\to\infty} e^x=\infty \quad , \quad \lim_{x\to -\infty}\frac{e^x}{x}=0$$

$x=0$ düşey asimptot, $y=0$ yatay asimptot

$$m=\lim_{x\to\infty}\frac{\frac{e^x}{x}}{x}=\lim_{x\to\infty}\frac{e^x}{x^2}=\lim_{x\to\infty}\frac{e^x}{2x}=\lim_{x\to\infty}\frac{e^x}{2}=\infty$$

Eğik asimptot yok.

$$f'(x)=\frac{e^x\,x-e^x\cdot 1}{x^2}=\frac{e^x\,(x-1)}{x^2} \Rightarrow$$

pozitif

$x=1$ Kritik Nokta

x		1	
$x-1$	$-$	0	$+$
f'	$-$	0	$+$

f, $(-\infty,1)$ de azalan, $(1,\infty)$ da artandır.

$x=1$ yerel min. nokta. $f(1)=e$

$$f''(x)=\frac{\left(e^x(x-1)+e^x\cdot 1\right)\cdot x^2-e^x(x-1)\cdot 2x}{x^4}=\frac{x\cdot e^x\left(x(x-1)+x-2x+2\right)}{x^4}$$

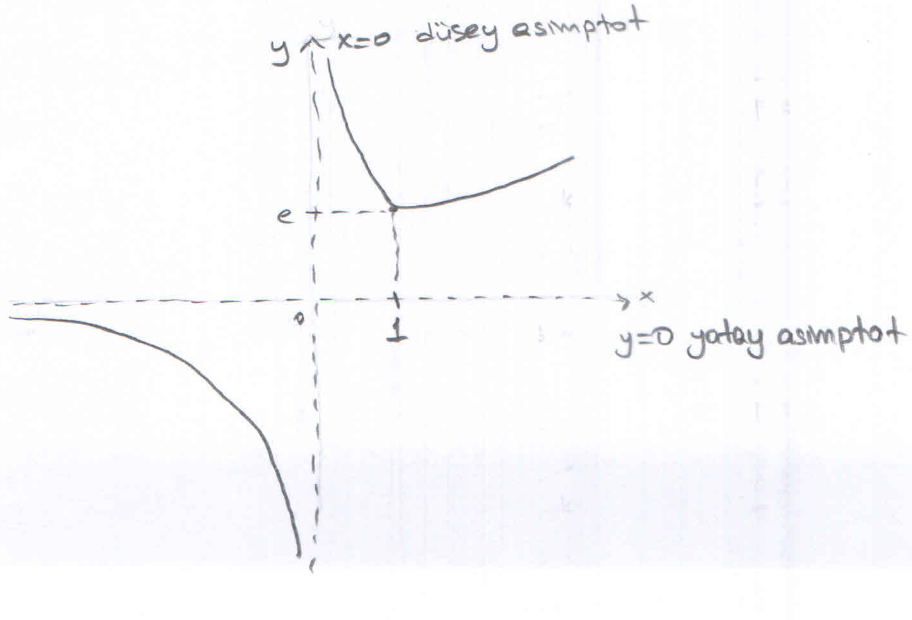
$$=\frac{e^x\cdot\left(x^2-x-x+2\right)}{x^3}=\frac{e^x\cdot\left(x^2-2x+2\right)}{x^3}=\frac{e^x\left((x-1)^2+1\right)}{x^3}$$

pozitif

$x=0$ Kritik Nokta

x		0	
x^3	$-$	0	$+$
$f''(x)$	$-$		$+$

f, $(-\infty,0)$ da konkav, $(0,\infty)$ da konvekstir



4) a) $\int \frac{\cot^2 x}{\sin^2 x}\, dx = ?$ b) $\int x^2 \sqrt{1+x}\, dx = ?$

Çözüm: a) $\int \frac{\cot^2 x}{\sin^2 x}\, dx = \int \cot^2 x \operatorname{cosec}^2 x\, dx = -\int u^2 du = -\frac{1}{3}\cdot \cot^3 x + c$

$(u = \cot x \Rightarrow du = -\operatorname{cosec}^2 x\, dx)$

b) $\int x^2 \sqrt{1+x}\, dx = \int (u-1)^2 u^{1/2}\, du = \int u^{5/2} - 2u^{3/2} + u^{1/2} du$

$\left(u = 1+x \Rightarrow \begin{array}{l} x = u-1 \\ du = dx \end{array} \right)$

$= \frac{2}{7} u^{7/2} - \frac{4}{5} u^{5/2} + \frac{2}{3} u^{3/2} + c$

$= \frac{2}{7} (x+1)^{7/2} - \frac{4}{5} (x+1)^{5/2} + \frac{2}{3} (x+1)^{3/2} + c$

5) a) $\int \ln(x^2+4)\, dx = ?$ b) $\int \sin^3 x \cos^3 x\, dx = ?$

Çözüm: a) $\int \ln(x^2+4)\, dx = x\ln(x^2+4) - 2\int \frac{x^2}{x^2+4} dx$

$$\boxed{\begin{array}{ll} u = \ln(x^2+4) & dv = dx \\ du = \frac{2x}{x^2+4} & v = x \end{array}}$$

$\int \frac{x^2}{x^2+4} dx = \int \frac{x^2+4-4}{x^2+4} dx = \int 1 - \frac{4}{x^2+4} dx$

$= \int dx - \int \frac{4}{x^2+4} dx$

$= x - 2\arctan\frac{x}{2} + c_1$

$\int \ln(x^2+4)\, dx = x\ln(x^2+4) - 2x + 4\arctan\frac{x}{2} + c$

b) $\int \sin^3 x \cos^3 x\, dx = \int \sin^3 x\, (1-\sin^2 x) \cos x\, dx$

$= \int \sin^3 x \cos x\, dx - \int \sin^5 x \cos x\, dx$

$$\boxed{\begin{array}{l} u = \sin x \\ du = \cos x\, dx \end{array}}$$

$= \int u^3 du - \int u^5 du$

$= \frac{u^4}{4} - \frac{u^6}{6} + c = \frac{1}{4}\sin^4 x - \frac{1}{6}\sin^6 x + c$